

Package ‘rbbnp’

March 26, 2025

Type Package

Title A Bias Bound Approach to Non-Parametric Inference

Version 0.2.0

Maintainer Xinyu DAI <xinyu_dai@brown.edu>

Description A novel bias-bound approach for non-parametric inference is introduced, focusing on both density and conditional expectation estimation.

It constructs valid confidence intervals that account for the presence of a non-negligible bias and thus make it possible to perform inference with optimal mean squared error minimizing bandwidths.

This package is based on Schennach (2020) <[doi:10.1093/restud/rdz065](https://doi.org/10.1093/restud/rdz065)>.

License GPL (>= 3)

Encoding UTF-8

LazyData true

URL <https://doi.org/10.1093/restud/rdz065>

Imports purrr, pracma, tidyr, dplyr, ggplot2, gridExtra

RoxygenNote 7.3.2

Depends R (>= 3.5)

NeedsCompilation no

Author Xinyu DAI [aut, cre],
Susanne M Schennach [aut]

Repository CRAN

Date/Publication 2025-03-26 17:40:02 UTC

Contents

biasBound_condExpectation	2
biasBound_density	4
create_biasBound_config	6
create_kernel_functions	7
DATA_PATH	8

epanechnikov_kernel	8
epanechnikov_kernel_ft	9
EXT_DATA_PATH	9
fun_approx	10
gen_sample_data	11
get_avg_f1x	11
get_avg_fyx	12
get_avg_phi	12
get_avg_phi_log	13
get_conditional_var	13
get_est_Ar	14
get_est_B	14
get_est_b1x	15
get_est_byx	15
get_est_vy	16
get_sigma	16
get_sigma_yx	17
get_xi_interval	17
kernel_reg	18
normal_kernel	19
normal_kernel_ft	19
plot_ft	20
rpoly01	20
sample_data	21
sinc	21
sinc_ft	22
true_density_2fold	22
W_kernel	23
W_kernel_ft	23

Index **24**

biasBound_condExpectation

Bias bound approach for conditional expectation estimation

Description

Estimates the density at a given point or across a range, and provides visualization options for density, bias, and confidence intervals.

Usage

```
biasBound_condExpectation(
  Y,
  X,
  x = NULL,
  h = 0.09,
```

```

alpha = 0.05,
est_Ar = NULL,
resol = 100,
xi_lb = NULL,
xi_ub = NULL,
methods_get_xi = "Schennach",
if_plot_ft = FALSE,
ora_Ar = NULL,
if_plot_conditional_mean = TRUE,
kernel.fun = "Schennach2004",
if_approx_kernel = TRUE,
kernel.resol = 1000
)

```

Arguments

Y	A numerical vector of sample data.
X	A numerical vector of sample data.
x	Optional. A scalar or range of points where the density is estimated. If NULL, a range is automatically generated.
h	A scalar bandwidth parameter.
alpha	Confidence level for intervals. Default is 0.05.
est_Ar	Optional list of estimates for A and r. If NULL, they are computed using <code>get_est_Ar()</code> .
resol	Resolution for the estimation range. Default is 100.
xi_lb	Optional. Lower bound for the interval of Fourier Transform frequency xi. Used for determining the range over which A and r is estimated. If NULL, it is automatically determined based on the <code>methods_get_xi</code> .
xi_ub	Optional. Upper bound for the interval of Fourier Transform frequency xi. Similar to <code>xi_lb</code> , it defines the upper range for A and r estimation. If NULL, the upper bound is determined based on the <code>methods_get_xi</code> .
methods_get_xi	A string specifying the method to automatically determine the xi interval if <code>xi_lb</code> and <code>xi_ub</code> are NULL. Options are "Schennach" and "Schennach_loose". If "Schennach" the range is selected based on the Theorem 2 in Schennach2020, if "Schennach_loose", it is defined by the initial interval given in Theorem 2 without selecting the <code>xi_n</code> .
if_plot_ft	Logical. If TRUE, plots the Fourier transform.
ora_Ar	Optional list of oracle values for A and r.
if_plot_conditional_mean	Logical. If TRUE, plots the conditional mean estimation.
kernel.fun	A string specifying the kernel function to be used. Options are "Schennach2004", "sinc", "normal", "epanechnikov".
if_approx_kernel	Logical. If TRUE, uses approximations for the kernel function.
kernel.resol	The resolution for kernel function approximation. See fun_approx .

Value

A list containing various outputs including estimated values, plots, and intervals.

Examples

```
# Example 1: point estimation of conditional expectation of Y on X
biasBound_condExpectation(
  Y = sample_data$Y,
  X = sample_data$X,
  x = 1,
  h = 0.09,
  kernel.fun = "Schennach2004"
)

# Example 2: conditional expectation of Y on X with manually selected range of xi
# biasBound_condExpectation(
# Y = sample_data$Y,
# X = sample_data$X,
# h = 0.09,
# xi_lb = 1,
# xi_ub = 12,
# kernel.fun = "Schennach2004"
# )
```

biasBound_density

Bias bound approach for density estimation

Description

Estimates the density at a given point or across a range, and provides visualization options for density, bias, and confidence intervals.

Usage

```
biasBound_density(
  X,
  x = NULL,
  h = 0.09,
  alpha = 0.05,
  resol = 100,
  xi_lb = NULL,
  xi_ub = NULL,
  methods_get_xi = "Schennach",
  if_plot_density = TRUE,
  if_plot_ft = FALSE,
  ora_Ar = NULL,
  kernel.fun = "Schennach2004",
```

```

    if_approx_kernel = TRUE,
    kernel.resol = 1000
)

```

Arguments

X	A numerical vector of sample data.
x	Optional. A scalar or range of points where the density is estimated. If NULL, a range is automatically generated.
h	A scalar bandwidth parameter.
alpha	Confidence level for intervals. Default is 0.05.
resol	Resolution for the estimation range. Default is 100.
xi_lb	Optional. Lower bound for the interval of Fourier Transform frequency xi. Used for determining the range over which A and r is estimated. If NULL, it is automatically determined based on the methods_get_xi.
xi_ub	Optional. Upper bound for the interval of Fourier Transform frequency xi. Similar to xi_lb, it defines the upper range for A and r estimation. If NULL, the upper bound is determined based on the methods_get_xi.
methods_get_xi	A string specifying the method to automatically determine the xi interval if xi_lb and xi_ub are NULL. Options are "Schennach" and "Schennach_loose". If "Schennach" the range is selected based on the Theorem 2 in Schennach2020, if "Schennach_loose", it is defined by the initial interval given in Theorem 2 without selecting the xi_n.
if_plot_density	Logical. If TRUE, plots the density estimation.
if_plot_ft	Logical. If TRUE, plots the Fourier transform.
ora_Ar	Optional list of oracle values for A and r.
kernel.fun	A string specifying the kernel function to be used. Options are "Schennach2004", "sinc", "normal", "epanechnikov".
if_approx_kernel	Logical. If TRUE, uses approximations for the kernel function.
kernel.resol	The resolution for kernel function approximation. See fun_approx .

Value

A list containing various outputs including estimated values, plots, and intervals.

Examples

```

# Example 1: Specifying x for point estimation with manually selected xi range from 1 to 12
biasBound_density(
  X = sample_data$X,
  x = 1,
  h = 0.09,
  xi_lb = 1,
  xi_ub = 12,

```

```

    if_plot_ft = TRUE,
    kernel.fun = "Schennach2004"
  )

# Example 2: Density estimation with manually selected xi range from 1 to 12 xi_lb and xi_ub
# biasBound_density(
#   X = sample_data$X,
#   h = 0.09,
#   xi_lb = 1,
#   xi_ub = 12,
#   if_plot_ft = FALSE,
#   kernel.fun = "Schennach2004"
# )

# Example 3: Density estimation with automatically selected xi range via Theorem 2 in Schennach 2020
# biasBound_density(
#   X = sample_data$X,
#   h = 0.09,
#   methods_get_xi = "Schennach",
#   if_plot_ft = TRUE,
#   kernel.fun = "Schennach2004"
# )

```

```
create_biasBound_config
```

Create a configuration object for bias bound estimations

Description

Create a configuration object for bias bound estimations

Usage

```

create_biasBound_config(
  X,
  Y = NULL,
  h = 0.09,
  alpha = 0.05,
  resol = 100,
  xi_lb = NULL,
  xi_ub = NULL,
  methods_get_xi = "Schennach",
  kernel.fun = "Schennach2004",
  if_approx_kernel = TRUE,
  kernel.resol = 1000
)

```

Arguments

X	A numerical vector of sample data.
Y	Optional. A numerical vector of sample data for conditional expectation.
h	A scalar bandwidth parameter.
alpha	Confidence level for intervals.
resol	Resolution for the estimation range.
xi_lb	Lower bound for the interval of Fourier Transform frequency.
xi_ub	Upper bound for the interval of Fourier Transform frequency.
methods_get_xi	Method to determine xi interval.
kernel.fun	Kernel function to be used.
if_approx_kernel	Use approximations for the kernel function.
kernel.resol	Resolution for kernel approximation.

Value

A configuration object (list) with all parameters

create_kernel_functions

Create kernel functions based on configuration

Description

Create kernel functions based on configuration

Usage

```
create_kernel_functions(
  kernel.fun = "Schennach2004",
  if_approx_kernel = TRUE,
  kernel.resol = 1000,
  X = NULL,
  h = 0.09
)
```

Arguments

kernel.fun	A string specifying the kernel function to be used.
if_approx_kernel	Logical. If TRUE, uses approximations for the kernel function.
kernel.resol	The resolution for kernel function approximation.
X	Optional data vector used to determine approximation range.
h	Optional bandwidth parameter used for approximation range.

Value

A list containing kernel function and its Fourier transform

DATA_PATH	<i>The Path to the Data Folder</i>
-----------	------------------------------------

Description

This variable provides the path to the data folder within the package.

Value

The path to the package's internal data folder as a character string.

epanechnikov_kernel	<i>Epanechnikov Kernel</i>
---------------------	----------------------------

Description

Epanechnikov Kernel

Usage

```
epanechnikov_kernel(u)
```

Arguments

u A numerical value or vector representing the input to the kernel function.

Value

Returns the value of the Epanechnikov kernel function at the given input.

`epanechnikov_kernel_ft`*Fourier Transform Epanechnikov Kernel*

Description

Fourier Transform Epanechnikov Kernel

Usage

```
epanechnikov_kernel_ft(xi)
```

Arguments

`xi` A numerical value or vector representing the frequency domain.

Value

Returns the value of the Fourier transform of the Epanechnikov kernel at the given frequency/frequencies.

`EXT_DATA_PATH`*The Path to the External Data Folder for Non-R Data Files*

Description

This variable provides the path to the `extdata` folder within the package, where non-standard R data files are stored.

Value

The path to the package's external data folder (for non-standard R data files) as a character string.

`fun_approx`*Approximation Function for Intensive Calculations*

Description

This function provides a lookup-based approximation for calculations that are computationally intensive. Once computed, it stores the results in an environment and uses linear interpolation for new data points to speed up subsequent computations.

Usage

```
fun_approx(u, u_lb = -100, u_ub = 100, resol = 1000, fun = W_kernel)
```

Arguments

<code>u</code>	A vector of values where the function should be evaluated.
<code>u_lb</code>	Lower bound for the precomputed range. Defaults to -10.
<code>u_ub</code>	Upper bound for the precomputed range. Defaults to 10.
<code>resol</code>	The resolution or number of sample points in the precomputed range. Defaults to 1000.
<code>fun</code>	A function for which the approximation is computed. Defaults to the W function.

Details

The `fun_approx` function works by initially creating a lookup table of function values based on the range specified by `u_lb` and `u_ub` and the resolution `resol`. This precomputation only happens once for a given set of parameters (`u_lb`, `u_ub`, `resol`, and `fun`). Subsequent calls to `fun_approx` with the same parameters use the lookup table to find the closest precomputed points to the requested `u` values and then return an interpolated result.

Linear interpolation is used between the two closest precomputed points in the lookup table. This ensures a smooth approximation for values in between sample points.

This function is especially useful for computationally intensive functions where recalculating function values is expensive or time-consuming. By using a combination of precomputation and interpolation, `fun_approx` provides a balance between accuracy and speed.

Value

A vector of approximated function values corresponding to `u`.

gen_sample_data	<i>Generate Sample Data</i>
-----------------	-----------------------------

Description

This function used for generate some sample data for experiment

Usage

```
gen_sample_data(size, dgp, seed = NULL)
```

Arguments

size	control the sample size.
dgp	data generating process, have options "normal", "chisq", "mixed", "poly", "2_fold_uniform".
seed	random seed number.

Value

A numeric vector of length size. The elements of the vector are generated according to the specified dgp:

normal Normally distributed values with mean 0 and standard deviation 2.

chisq Chi-squared distributed values with $df = 10$.

mixed Half normally distributed (mean 0, $sd = 2$) and half chi-squared distributed ($df = 10$) values.

poly Values from a polynomial cumulative distribution function on $[\theta, 1]$.

2_fold_uniform Sum of two uniformly distributed random numbers.

get_avg_f1x	<i>Kernel point estimation</i>
-------------	--------------------------------

Description

Computes the point estimate using the specified kernel function.

Usage

```
get_avg_f1x(X, x, h, inf_k)
```

Arguments

X	A numerical vector of sample data.
x	A scalar representing the point where the density is estimated.
h	A scalar bandwidth parameter.
inf_k	Kernel function used for the computation.

Value

A scalar representing the kernel density estimate at point x .

get_avg_fyx	<i>Kernel point estimation</i>
-------------	--------------------------------

Description

Computes the point estimate using the specified kernel function.

Usage

```
get_avg_fyx(Y, X, x, h, inf_k)
```

Arguments

Y	A numerical vector representing the sample data of variable Y.
X	A numerical vector representing the sample data of variable X.
x	A scalar representing the point where the density is estimated.
h	A scalar bandwidth parameter.
inf_k	Kernel function used for the computation.

Value

A scalar representing the kernel density estimate at point x .

get_avg_phi	<i>Compute Sample Average of Fourier Transform Magnitude</i>
-------------	--

Description

Compute Sample Average of Fourier Transform Magnitude

Usage

```
get_avg_phi(Y = 1, X, xi)
```

Arguments

Y	A numerical vector representing the sample data of variable Y.
X	A numerical vector representing the sample data of variable X.
xi	A single numerical value representing the frequency at which the Fourier transform is computed.

Value

Returns the sample estimation of expected Fourier transform at frequency ξ .

get_avg_phi_log *Compute log sample average of fourier transform and get mod*

Description

Compute log sample average of fourier transform and get mod

Usage

```
get_avg_phi_log(Y = 1, X, ln_xi)
```

Arguments

Y	A numerical vector representing the sample data of variable Y.
X	A numerical vector representing the sample data of variable X.
ln_xi	A single numerical value representing the log frequency at which the Fourier transform is computed.

Value

Returns the log sample estimation of expected Fourier transform at frequency xi.

get_conditional_var *get the conditional variance of Y on X for given x*

Description

get the conditional variance of Y on X for given x

Usage

```
get_conditional_var(X, Y, x, h, kernel_func)
```

Arguments

X	A numerical vector representing the sample data of variable X.
Y	A numerical vector representing the sample data of variable Y.
x	The specific point at which the conditional variance is to be calculated.
h	A bandwidth parameter used in the kernel function for smoothing.
kernel_func	A kernel function used to weigh observations in the neighborhood of point x.

Value

Returns a non-negative scalar representing the estimated conditional variance of Y given X at the point x. Returns 0 if the computed variance is negative.

get_est_Ar *get the estimation of A and r*

Description

This function estimates the parameters A and r by optimizing an objective function over a specified range of frequency values and r values.

Usage

```
get_est_Ar(Y = 1, X, xi_interval, r_stepsize = 150)
```

Arguments

Y	A numerical vector representing the sample data of variable Y.
X	A numerical vector representing the sample data of variable X.
xi_interval	A list with elements xi_lb and xi_ub representing the lower and upper bounds of the frequency interval.
r_stepsize	An integer value representing the number of steps in the r range. This controls the granularity of the estimation. Higher values lead to finer granularity but increase computation time.

Details

The function internally defines a range for the natural logarithm of frequency values (`ln_xi_range`) and a range for the parameter `r` (`r_range`). It then defines an optimization function `optim_ln_A` to minimize the integral of a given function over the `ln_xi_range`. The actual estimation is done by finding the `r` and `A` value that minimizes the the area of the line $\ln A - r \ln \xi$ under the constraint that the line should not go below the Fourier transform curve.

Value

A named vector with elements `est_A` and `est_r` representing the estimated values of A and r, respectively.

get_est_B *get the estimation of B*

Description

get the estimation of B

Usage

```
get_est_B(Y)
```

Arguments

Y A numerical vector representing the sample data of variable Y.

Value

The mean of the absolute values of the elements in Y, representing the estimated value of B .

get_est_b1x *Estimation of bias b1x*

Description

Computes the bias estimate for given parameters.

Usage

get_est_b1x(X, ...)

Arguments

X A numerical vector representing the sample data of variable X.
 ... Additional arguments passed to other methods.

Value

A scalar representing the bias b1x estimate.

get_est_byx *Estimation of bias byx*

Description

Estimation of bias byx

Usage

get_est_byx(Y, X, ...)

Arguments

Y A numerical vector representing the sample data of variable Y.
 X A numerical vector representing the sample data of variable X.
 ... Additional arguments passed to other methods.

Value

A scalar representing the bias byx estimate.

get_est_vy	<i>get the estimation of Vy</i>
------------	---------------------------------

Description

get the estimation of Vy

Usage

```
get_est_vy(Y)
```

Arguments

Y	A numerical vector representing the sample data of variable Y.
---	--

get_sigma	<i>Estimation of sigma</i>
-----------	----------------------------

Description

Computes the sigma estimate for given parameters.

Usage

```
get_sigma(X, x, h, inf_k)
```

Arguments

X	A numerical vector of sample data.
x	A scalar representing the point where the density is estimated.
h	A scalar bandwidth parameter.
inf_k	Kernel function used for the computation.

Value

A scalar representing the sigma estimate at point x. Returns 0 if the density estimate is negative.

get_sigma_yx	<i>Estimation of sigma_yx</i>
--------------	-------------------------------

Description

Estimation of sigma_yx

Usage

```
get_sigma_yx(Y, X, x, h, inf_k)
```

Arguments

Y	A numerical vector representing the sample data of variable Y.
X	A numerical vector representing the sample data of variable X.
x	The specific point at which sigma_yx is to be estimated.
h	A bandwidth parameter used in the kernel function for smoothing.
inf_k	A kernel function used to weigh observations in the neighborhood of point x.

Value

Returns a scalar representing the estimated value of sigma_yx at the point x. Returns 0 if either fyx or conditional variance is negative.

get_xi_interval	<i>get xi interval</i>
-----------------	------------------------

Description

get xi interval

Usage

```
get_xi_interval(Y = 1, X, methods = "Schennach")
```

Arguments

Y	A numerical vector representing the sample data of variable Y.
X	A numerical vector representing the sample data of variable X.
methods	A character string indicating the method to use for calculating the xi interval. Supported methods are "Schennach" and "Schennach_loose". Defaults to "Schennach".

Details

The "Schennach" method computes the xi interval by performing a test based on the Schennach's theorem, adjusting the upper bound `xi_ub` if the test condition is met. The "Schennach_loose" method provides a looser calculation of the xi interval without performing the Schennach's test.

Value

A list containing the lower (`xi_lb`) and upper (`xi_ub`) bounds of the xi interval.

<code>kernel_reg</code>	<i>Kernel Regression function</i>
-------------------------	-----------------------------------

Description

Kernel Regression function

Usage

```
kernel_reg(X, Y, x, h, kernel_func)
```

Arguments

<code>X</code>	A numerical vector representing the sample data of variable X.
<code>Y</code>	A numerical vector representing the sample data of variable Y.
<code>x</code>	The point at which the regression function is to be estimated.
<code>h</code>	A bandwidth parameter that determines the weight assigned to each observation in X.
<code>kernel_func</code>	A function that computes the weight of each observation based on its distance to x.

Value

Returns a scalar representing the estimated value of the regression function at the point x.

normal_kernel	<i>Normal Kernel Function</i>
---------------	-------------------------------

Description

Normal Kernel Function

Usage

normal_kernel(u)

Arguments

u A numerical value or vector representing the input to the kernel function.

Value

Returns the value of the Normal kernel function at the given input.

normal_kernel_ft	<i>Fourier Transform of Normal Kernel</i>
------------------	---

Description

Fourier Transform of Normal Kernel

Usage

normal_kernel_ft(xi)

Arguments

xi A numerical value or vector representing the frequency domain.

Value

Returns the value of the Fourier transform of the Normal kernel at the given frequency/frequencies.

`plot_ft`*Plot the Fourier Transform*

Description

Plot the Fourier Transform of the

Usage

```
plot_ft(X, xi_interval, ft_plot.resol = 500)
```

Arguments

`X` A numerical vector of sample data.

`xi_interval` A list containing the lower (`xi_lb`) and upper (`xi_ub`) bounds of the `xi` interval.

`ft_plot.resol` An integer representing the resolution of the plot, specifically the number of points used to represent the Fourier transform. Defaults to 500.

Details

$C = 1$, the parameter in $O(1/n^{0.25})$, see more details in Schennach (2020).

Value

A ggplot object representing the plot of the Fourier transform.

Examples

```
plot_ft(  
  sample_data$X,  
  xi_interval = list(xi_lb = 1, xi_ub = 50),  
  ft_plot.resol = 1000  
)
```

`rpoly01`*Generate n samples from the distribution*

Description

Generate n samples from the distribution

Usage

```
rpoly01(n, k = 5)
```

Arguments

- n The number of samples to generate.
 k The exponent in the distribution function, defaults to 5.

Value

A vector of n samples from the specified polynomial distribution.
 CDF: $f(x) = (x-1)^k + 1$

sample_data	<i>Sample Data</i>
-------------	--------------------

Description

Sample Data

Usage

sample_data

Format

A data frame with 1000 rows and 2 variables:
X Numeric vector, generated from 2 fold uniform distribution.
Y Numeric vector, $Y = -X^2 + 3*X + \text{rnorm}(1000)*X$.

sinc	<i>Infinite Kernel Function</i>
------	---------------------------------

Description

Infinite Kernel Function

Usage

sinc(u)

Arguments

- u A numerical value or vector where the sinc function is evaluated.

Value

The value of the sinc function at each point in u.

`sinc_ft`*Define the closed form FT of the infinite order kernel $\sin(x)/(\pi*x)$*

Description

Define the closed form FT of the infinite order kernel $\sin(x)/(\pi*x)$

Usage`sinc_ft(x)`**Arguments**

`x` A numerical value or vector where the Fourier Transform is evaluated.

Value

The value of the Fourier Transform of the sinc function at each point in `x`.

`true_density_2fold`*True density of 2-fold uniform distribution*

Description

True density of 2-fold uniform distribution

Usage`true_density_2fold(x)`**Arguments**

`x` A numerical value or vector where the true density function is evaluated.

Value

The value of the true density of the 2-fold uniform distribution at each point in `x`.

W_kernel	<i>Define the inverse Fourier transform function of W</i>
----------	---

Description

Define the inverse Fourier transform function of W

Usage

W_kernel(u, L = 10)

Arguments

u	A numerical value or vector representing the time or space domain.
L	The limit for numerical integration, defines the range of integration as $[-L, L]$. Defaults to 10.

Value

A numerical value or vector representing the inverse Fourier transform of the infinite order kernel at the given time or space point(s).

W_kernel_ft	<i>Define the Fourier transform of a infinite kernel proposed in Schennach 2004</i>
-------------	---

Description

Define the Fourier transform of a infinite kernel proposed in Schennach 2004

Usage

W_kernel_ft(xi, xi_lb = 0.5, xi_ub = 1.5)

Arguments

xi	A numerical value or vector representing the frequency domain.
xi_lb	The lower bound for the frequency domain. Defaults to 0.5.
xi_ub	The upper bound for the frequency domain. Defaults to 1.5.

Value

A numerical value or vector representing the Fourier transform of the infinite order kernel at the given frequency/frequencies.

Index

* datasets

sample_data, 21

biasBound_condExpectation, 2

biasBound_density, 4

create_biasBound_config, 6

create_kernel_functions, 7

DATA_PATH, 8

epanechnikov_kernel, 8

epanechnikov_kernel_ft, 9

EXT_DATA_PATH, 9

fun_approx, 3, 5, 10

gen_sample_data, 11

get_avg_f1x, 11

get_avg_fyx, 12

get_avg_phi, 12

get_avg_phi_log, 13

get_conditional_var, 13

get_est_Ar, 14

get_est_B, 14

get_est_b1x, 15

get_est_byx, 15

get_est_vy, 16

get_sigma, 16

get_sigma_yx, 17

get_xi_interval, 17

kernel_reg, 18

normal_kernel, 19

normal_kernel_ft, 19

plot_ft, 20

rpoly01, 20

sample_data, 21

sinc, 21

sinc_ft, 22

true_density_2fold, 22

W_kernel, 23

W_kernel_ft, 23