

Package ‘einsum’

August 28, 2023

Type Package

Title Einstein Summation

Version 0.1.2

Description

The summation notation suggested by Einstein (1916) <[doi:10.1002/andp.19163540702](https://doi.org/10.1002/andp.19163540702)> is a concise mathematical notation that implicitly sums over repeated indices of n-dimensional arrays. Many ordinary matrix operations (e.g. transpose, matrix multiplication, scalar product, 'diag()', trace etc.) can be written using Einstein notation. The notation is particularly convenient for expressing operations on arrays with more than two dimensions because the respective operators ('tensor products') might not have a standardized name.

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Encoding UTF-8

Suggests testthat,
covr

RdMacros mathjaxr

RoxygenNote 7.2.3

LinkingTo Rcpp

Imports Rcpp,
glue,
mathjaxr

URL <https://const-ae.github.io/einsum/>

BugReports <https://github.com/const-ae/einsum/issues>

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einsum

*Einstein Summation***Description**

Einstein summation is a convenient and concise notation for operations on n-dimensional arrays.

Usage

```
einsum(equation_string, ...)
```

```
einsum_generator(equation_string, compile_function = TRUE)
```

Arguments

equation_string

a string in Einstein notation where arrays are separated by ',' and the result is separated by '->'. For example "ij,jk->ik" corresponds to a standard matrix multiplication. Whitespace inside the equation_string is ignored. Unlike the equivalent functions in Python, einsum() only supports the explicit mode. This means that the equation_string must contain '->'.
 ... the arrays that are combined. All arguments are converted to arrays with as.array.
 compile_function boolean that decides if einsum_generator() returns the result of Rcpp::cppFunction() or the program as a string. Default: TRUE.

Details

The following table show, how the Einstein notation abbreviates complex summation for arrays/matrices:

equation_string	Formula	
"ij,jk->ik"	$Y_{ik} = \sum_j A_{ij} B_{jk}$	Matrix multiplication
"ij->ji"	$Y = A^T$	Transpose
"ii->i"	$y = \text{diag}(A)$	Diagonal
"ii->ii"	$Y = \text{diag}(A)I$	Diagonal times Identity
"ii->"	$y = \text{trace}(A) = \sum_i A_{ii}$	Trace
"ijk,mjj->i"	$y_i = \sum_j \sum_k \sum_m A_{ijk} B_{mjj}$	Complex 3D operation

The function and the conventions are inspired by the einsum() function in NumPy ([documentation](#)). Unlike NumPy, 'einsum' only supports the explicit mode. The explicit mode is more flexible and can avoid confusion. The common summary of the Einstein summation to "sum over duplicated indices" however is not a good mental model. A better rule of thumb is "sum over all indices not in the result".

Note: einsum() internally uses C++ code to provide results quickly, the repeated parsing of the equation_string comes with some overhead. Thus, if you need to do the same calculation over and over again it can be worth to use einsum_generator() and call the returned the function. einsum_generator() generates efficient C++ code that can be one or two orders of magnitude faster than einsum().

Value

The `einsum()` function returns an array with one dimension for each index in the result of the `equation_string`. For example `"ij,jk->ik"` produces a 2-dimensional array, `"abc,cd,de->abe"` produces a 3-dimensional array.

The `einsum_generator()` function returns a function that takes one array for each comma-separated input in the `equation_string` and returns the same result as `einsum()`. Or if `compile_function = FALSE`, `einsum_generator()` function returns a string with the C++ code for such a function.

Examples

```
mat1 <- matrix(rnorm(n = 4 * 8), nrow = 4, ncol = 8)
mat2 <- matrix(rnorm(n = 8 * 3), nrow = 8, ncol = 3)

# Matrix Multiply
mat1 %*% mat2
einsum("ij,jk -> ik", mat1, mat2)

# einsum_generator() works just like einsum() but returns a performant function
mat_mult <- einsum_generator("ij,jk -> ik")
mat_mult(mat1, mat2)

# Diag
mat_sq <- matrix(rnorm(n = 4 * 4), nrow = 4, ncol = 4)
diag(mat_sq)
einsum("ii->i", mat_sq)
einsum("ii->ii", mat_sq)

# Trace
sum(diag(mat_sq))
einsum("ii->", mat_sq)

# Scalar product
mat3 <- matrix(rnorm(n = 4 * 8), nrow = 4, ncol = 8)
mat3 * mat1
einsum("ij,ij->ij", mat3, mat1)

# Transpose
t(mat1)
einsum("ij->ji", mat1)

# Batched L2 norm
arr1 <- array(c(mat1, mat3), dim = c(dim(mat1), 2))
c(sum(mat1^2), sum(mat3^2))
einsum("ijb,ijb->b", arr1, arr1)
```

Description

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