

# Package ‘SANvi’

January 20, 2025

**Type** Package

**Title** Fitting Shared Atoms Nested Models via Variational Bayes

**Version** 0.1.1

**Date** 2024-05-14

**Maintainer** Francesco Denti <francescodenti.personal@gmail.com>

**URL** <https://github.com/fradenti/SANvi>

**BugReports** <https://github.com/fradenti/SANvi/issues>

**Description** An efficient tool for fitting the nested common and shared atoms models using variational Bayes approximate inference for fast computation. Specifically, the package implements the common atoms model (Denti et al., 2023), its finite version (D’Angelo et al., 2023), and a hybrid finite-infinite model.

All models use Gaussian mixtures with a normal-inverse-gamma prior distribution on the parameters. Additional functions are provided to help analyze the results of the fitting procedure.

References:

Denti, Camerlenghi, Guindani, Mira (2023) <[doi:10.1080/01621459.2021.1933499](https://doi.org/10.1080/01621459.2021.1933499)>,

D’Angelo, Canale, Yu, Guindani (2023) <[doi:10.1111/biom.13626](https://doi.org/10.1111/biom.13626)>.

**License** MIT + file LICENSE

**Encoding** UTF-8

**RoxygenNote** 7.3.1

**Imports** Rcpp, matrixStats

**Depends** scales, RColorBrewer

**LinkingTo** Rcpp, RcppArmadillo

**Language** en-US

**NeedsCompilation** yes

**Author** Francesco Denti [aut, cre, cph]

(<<https://orcid.org/0000-0001-5034-7414>>),

Laura D’Angelo [aut] (<<https://orcid.org/0000-0003-2978-4702>>)

**Repository** CRAN

**Date/Publication** 2024-05-15 11:40:06 UTC

## Contents

estimate_atoms_weights_vi . . . . .	2
estimate_clustering_vi . . . . .	3
extract_best . . . . .	5
plot.SANvb . . . . .	5
print.SANvb . . . . .	6
variational_CAM . . . . .	7
variational_fiSAN . . . . .	10
variational_fSAN . . . . .	12
variational_multistart . . . . .	15

<b>Index</b>	<b>18</b>
--------------	-----------

---

estimate\_atoms\_weights\_vi

*Estimate the Posterior Atoms and Weights of the Discrete Mixing Distributions*

---

### Description

This function estimates the posterior atoms and weights characterizing the discrete mixing distributions using the variational estimates obtained from one of the model implemented in SANvi.

### Usage

```
estimate_atoms_weights_vi(output)

## S3 method for class 'vi_atoms_weights'
plot(x, DC_num = NULL, lim = 2, ...)

## S3 method for class 'vi_atoms_weights'
print(x, thr = 0.01, ...)
```

### Arguments

output	an object of class SANvb, which is the output of one of the variational functions <a href="#">variational_CAM</a> , <a href="#">variational_fiSAN</a> , <a href="#">variational_fSAN</a> .
x	an object of class <a href="#">variational_estimates</a> , which can be obtained from the function <a href="#">estimate_atoms_weights_vi</a> .
DC_num	an integer or a vector of integers indicating which distributional clusters to plot.
lim	optional value for plot method to adjust the limits of the x-axis. The atoms are plotted on a range given by $\min(\text{posterior means})-2$ , $\max(\text{posterior means})+2$ . Default is set to 2.
...	ignored.
thr	argument for the <code>print()</code> method. It should be a small positive number, representing a threshold. If the posterior weight of a shared atom is below the threshold, the atom is not reported.

**Value**

an object of class `vi_atoms_weights`, which is matrix comprising posterior means, variances, and a columns for each estimated DC containing the posterior weights.

**See Also**

[variational\\_CAM](#), [variational\\_fiSAN](#), [variational\\_fSAN](#), [extract\\_best](#).

**Examples**

```
# Generate example data
set.seed(1232)
y <- c(rnorm(100), rnorm(100, 5))
g <- rep(1:2, rep(100, 2))

# Fitting fiSAN via variational inference
est <- variational_fiSAN(y, g, verbose = FALSE)

# Estimate posterior atoms and weights
estimate_atoms_weights_vi(est)
```

---

```
estimate_clustering_vi
```

*Estimate Posterior Clustering Assignments*

---

**Description**

This function estimates posterior clustering assignments based on posterior variational estimates obtained from one of the model implemented in SANvi.

**Usage**

```
estimate_clustering_vi(output, ordered = TRUE)

## S3 method for class 'vi_clustering'
plot(
  x,
  DC_num = NULL,
  type = c("ecdf", "boxplot", "scatter"),
  palette_brewed = FALSE,
  ...
)

## S3 method for class 'vi_clustering'
print(x, ...)
```

**Arguments**

output	an object of class SANvb, the output of one of the variational functions <a href="#">variational_CAM</a> , <a href="#">variational_fiSAN</a> , <a href="#">variational_fSAN</a> .
ordered	logical, if TRUE (default), the function sorts the distributional cluster labels reflecting the increasing values of medians of the data assigned to each DC.
x	an object of class <code>variational_estimates</code> , which can be obtained from the function <a href="#">estimate_atoms_weights_vi</a> .
DC_num	an integer or a vector of integers indicating which distributional clusters to plot.
type	what type of plot should be drawn (only for the left-side plot). Possible types are "boxplot", "ecdf", and "scatter".
palette_brewed	(logical) the color palette to be used. Default is R base colors (palette_brewed = FALSE).
...	ignored.

**Value**

a list of class `clustering` containing

- `obs_level`: a data frame containing the data values, their group indexes, the observational and distributional clustering assignments for each observation.
- `dis_level`: a vector with the distributional clustering assignment for each unit.

**See Also**

[variational\\_CAM](#), [variational\\_fiSAN](#), [variational\\_fSAN](#), [extract\\_best](#).

**Examples**

```
# Generate example data
set.seed(123)
y <- c(rnorm(100), rnorm(100, -5), rnorm(100, 5), rnorm(100),
      rnorm(100), rnorm(100, -5), rnorm(100, 5), rnorm(100))
g <- rep(1:4, rep(200, 4))

# Fitting fiSAN via variational inference
est <- SANvi::variational_fiSAN(y, g, verbose = FALSE)

# Estimate clustering assignments
estimate_clustering_vi(est)
```

---

extract_best	<i>Extract the best run from multiple trials</i>
--------------	--

---

**Description**

A simple function to automatically extract the best run from a collection of fitted variational models.

**Usage**

```
extract_best(object)
```

**Arguments**

`object` an object of class `multistart`, obtained from the `variational_multistart` function.

**Value**

the single best run, an object of class `SANvb`.

**Examples**

```
# Generate example dataset
set.seed(123)
y <- c(rnorm(100), rnorm(100, 5))
g <- rep(1:2, rep(100, 2))

# Estimate multiple models via variational inference
est <- variational_multistart(y, g, runs=5,
                             alpha_bar = 3, beta_bar = 3,
                             root=1234, warmstart = FALSE)

# Obtain best run
extract_best(est)
```

---

plot.SANvb	<i>Plotting the variational inference output</i>
------------	--

---

**Description**

Plot method for objects of class `SANvb`. The function displays two graphs, meant to analyze the estimated distributional and observational clusters.

**Usage**

```
## S3 method for class 'SANvb'
plot(x, ...)
```

**Arguments**

- x object of class SANvb (the result of a call to [variational\\_CAM](#), [variational\\_fiSAN](#), [variational\\_fSAN](#)).
- ... additional graphical parameters to be passed.

**Value**

The function plots a summary of the fitted model.

**See Also**

[print.SANvb](#), [variational\\_CAM](#), [variational\\_fiSAN](#), [variational\\_fSAN](#).

---

<code>print.SANvb</code>	<i>Print variational inference output</i>
--------------------------	---

---

**Description**

Print method for objects of class SANvb.

**Usage**

```
## S3 method for class 'SANvb'  
print(x, ...)
```

**Arguments**

- x object of class SANvb (the result of a call to [variational\\_CAM](#), [variational\\_fiSAN](#), [variational\\_fSAN](#)).
- ... further arguments passed to or from other methods.

**Value**

The function prints a summary of the fitted model.

---

variational_CAM	<i>Mean Field Variational Bayes estimation of CAM</i>
-----------------	---

---

### Description

variational\_CAM is used to perform posterior inference under the common atoms model (CAM) of Denti et al. (2023) with Gaussian likelihood. The model uses Dirichlet process mixtures (DPM) at both the observational and distributional levels.

### Usage

```
variational_CAM(y, group, maxL = 30, maxK = 20,
               m0 = 0, tau0 = .01, lambda0 = 3, gamma0 = 2,
               conc_hyperpar = c(1,1,1,1), conc_par = NULL,
               epsilon = 1e-6, seed = NULL, maxSIM = 1e5,
               warmstart = TRUE, verbose = FALSE)
```

### Arguments

y	Numerical vector of observations (required).
group	Numerical vector of the same length of y, indicating the group membership (required).
maxL, maxK	integers, the upper bounds for the observational and distributional clusters to fit, respectively.
m0, tau0, lambda0, gamma0	Hyperparameters on $(\mu, \sigma^2) \sim NIG(m_0, \tau_0, \lambda_0, \gamma_0)$ .
conc_hyperpar, conc_par	Vectors of values used for the concentration parameters of of the stick-breaking representation for the distributional and observational DPs, respectively. The following two arguments can be passed. Specifically, conc_hyperpar a vector with 4 positive entries: $(s_1^\alpha, s_2^\alpha, s_1^\beta, s_2^\beta)$ . If a random concentration parameters $\alpha$ and $\beta$ are adopted, the specifications are $\alpha \sim Gamma(s_1^\alpha, s_2^\alpha)$ and $\beta \sim Gamma(s_1^\beta, s_2^\beta)$ . Default set to unitary vector. conc_par a vector with 2 positive entries: $(\alpha, \beta)$ . Default is set to NULL. If specified, the previous argument is ignored and the two concentration parameters are assumed fixed and equal to (alpha, beta).
epsilon	the tolerance that drives the convergence criterion adopted as stopping rule
seed	random seed to control the initialization.
maxSIM	the maximum number of CAVI iteration to perform.
warmstart	logical, if TRUE, the observational means of the cluster atoms are initialized with a k-means algorithm.
verbose	logical, if TRUE the iterations are printed.

## Details

The common atoms mixture model is used to perform inference in nested settings, where the data are organized into  $J$  groups. The data should be continuous observations  $(Y_1, \dots, Y_J)$ , where each  $Y_j = (y_{1,j}, \dots, y_{n_j,j})$  contains the  $n_j$  observations from group  $j$ , for  $j = 1, \dots, J$ . The function takes as input the data as a numeric vector  $y$  in this concatenated form. Hence  $y$  should be a vector of length  $n_1 + \dots + n_J$ . The group parameter is a numeric vector of the same size as  $y$  indicating the group membership for each individual observation. Notice that with this specification the observations in the same group need not be contiguous as long as the correspondence between the variables  $y$  and group is maintained.

### Model

The data are modeled using a Gaussian likelihood, where both the mean and the variance are observational-cluster-specific, i.e.,

$$y_{i,j} \mid M_{i,j} = l \sim N(\mu_l, \sigma_l^2)$$

where  $M_{i,j} \in \{1, 2, \dots\}$  is the observational cluster indicator of observation  $i$  in group  $j$ . The prior on the model parameters is a Normal-Inverse-Gamma distribution  $(\mu_l, \sigma_l^2) \sim NIG(m_0, \tau_0, \lambda_0, \gamma_0)$ , i.e.,  $\mu_l \mid \sigma_l^2 \sim N(m_0, \sigma_l^2/\tau_0)$ ,  $1/\sigma_l^2 \sim \text{Gamma}(\lambda_0, \gamma_0)$  (shape, rate).

### Clustering

The model performs a clustering of both observations and groups. The clustering of groups (distributional clustering) is provided by the allocation variables  $S_j \in \{1, 2, \dots\}$ , with

$$Pr(S_j = k \mid \dots) = \pi_k \quad \text{for } k = 1, 2, \dots$$

The distribution of the probabilities is  $\{\pi_k\}_{k=1}^{\infty} \sim GEM(\alpha)$ , where GEM is the Griffiths-Engen-McCloskey distribution of parameter  $\alpha$ , which characterizes the stick-breaking construction of the DP (Sethuraman, 1994).

The clustering of observations (observational clustering) is provided by the allocation variables  $M_{i,j} \in \{1, 2, \dots\}$ , with

$$Pr(M_{i,j} = l \mid S_j = k, \dots) = \omega_{l,k} \quad \text{for } k = 1, 2, \dots; l = 1, 2, \dots$$

The distribution of the probabilities is  $\{\omega_{l,k}\}_{l=1}^{\infty} \sim GEM(\beta)$  for all  $k = 1, 2, \dots$

## Value

variational\_CAM returns a list of class SANvb containing four objects:

- model: name of the fitted model.
- params: list containing the data and the parameters used in the simulation. Details below.
- sim: list containing the simulated values (optimized variational parameters). Details below.
- time: total computation time.

**Data and parameters:** params is a list with the following components:

y, group, Nj, J Data, group labels, group frequencies, and number of groups.

K, L Number of fitted distributional and observational clusters.



`m0`, `tau0`, `lambda0`, `gamma0` Model hyperparameters.

`epsilon`, `seed` The threshold controlling the convergence criterion and the random seed adopted to replicate the run.

`(hyp_alpha1, hyp_alpha2)` or `alpha` Hyperparameters on  $\alpha$  (if  $\alpha$  random); or provided value for  $\alpha$  (if fixed).

`(hyp_beta1, hyp_beta2)` or `beta` Hyperparameters on  $\beta$  (if  $\beta$  random); or provided value for  $\beta$  (if fixed).

**Simulated values:** `sim` is a list with the following components:

`theta_l` Matrix of size (L,4). Each row is a posterior variational estimate of the four normal-inverse gamma hyperparameters.

`Elbo_val` Vector containing the values of the ELBO.

`XI` A list of length J. Each element is a matrix of size (N, L) posterior variational probability of assignment of the i-th observation in the j-th group to the l-th OC, i.e.,  $\hat{\xi}_{i,j,l} = \hat{Q}(M_{i,j} = l)$ .

`RHO` Matrix of size (J, K). Each row is a posterior variational probability of assignment of the j-th group to the k-th DC, i.e.,  $\hat{\rho}_{j,k} = \hat{Q}(S_j = k)$ .

`a_tilde_k, b_tilde_k` Vector of updated variational parameters of the Beta distributions governing the distributional stick-breaking process.

`a_tilde_lk, b_tilde_lk` Matrix of updated variational parameters of the Beta distributions governing the observational stick-breaking process (arranged by column).

`conc_hyper` If the concentration parameters are chosen to be random, these object contain a vector with the four updated hyperparameters.

`alpha, beta` If the concentration parameters are chosen to be fixed, these objects contain the passed values.

## References

Denti, F., Camerlenghi, F., Guindani, M., and Mira, A. (2023). A Common Atoms Model for the Bayesian Nonparametric Analysis of Nested Data. *Journal of the American Statistical Association*, 118(541), 405-416. DOI: 10.1080/01621459.2021.1933499

Sethuraman, A.J. (1994). A Constructive Definition of Dirichlet Priors, *Statistica Sinica*, 4, 639–650.

## Examples

```
set.seed(123)
y <- c(rnorm(100), rnorm(100, 5))
g <- rep(1:2, rep(100, 2))
est <- variational_CAM(y, g, verbose = FALSE, epsilon = 1e-2)
```

---

variational\_fiSAN      *Mean Field Variational Bayes estimation of fiSAN*

---

## Description

variational\_fiSAN is used to perform posterior inference under the finite-infinite shared atoms nested (fiSAN) model with Gaussian likelihood. The model uses a Dirichlet process mixture prior at the distributional level, and finite Dirichlet mixture at the observational one.

## Usage

```
variational_fiSAN(y, group,
                 maxL = 30, maxK = 20,
                 m0 = 0, tau0 = .01, lambda0 = 3, gamma0 = 2,
                 conc_hyperpar = c(1,1), conc_par = NULL,
                 beta_bar = .005,
                 epsilon = 1e-6, seed = NULL,
                 maxSIM = 1e5, warmstart = TRUE, verbose = FALSE)
```

## Arguments

y	Numerical vector of observations (required).
group	Numerical vector of the same length of y, indicating the group membership (required).
maxL, maxK	integers, the upper bounds for the observational and distributional clusters to fit, respectively
m0, tau0, lambda0, gamma0	Hyperparameters on $(\mu, \sigma^2) \sim NIG(m_0, \tau_0, \lambda_0, \gamma_0)$ .
conc_hyperpar, conc_par	Vectors of values used the concentration parameters of of the stick-breaking representation for the distributional and observational DPs, respectively. The following two arguments can be passed. Specifically, conc_hyperpar a vector with 2 positive entries: $(s_1^\alpha, s_2^\alpha)$ . If a random concentration parameter $\alpha$ is adopted, the specification is $\alpha \sim Gamma(s_1^\alpha, s_2^\alpha)$ . Default set to unitary vector. conc_par a vector with one positive entry $\alpha$ . Default is set to NULL. If specified, the previous argument is ignored and the two concentration parameters are assumed fixed and equal to alpha.
beta_bar	the hyperparameter of the symmetric observational Dirichlet distribution.
epsilon	the tolerance that drives the convergence criterion adopted as stopping rule
seed	random seed to control the initialization.
maxSIM	the maximum number of CAVI iteration to perform.
warmstart	logical, if TRUE, the observational means of the cluster atoms are initialized with a k-means algorithm.
verbose	logical, if TRUE the iterations are printed.

## Details

### Data structure

The finite-infinite common atoms mixture model is used to perform inference in nested settings, where the data are organized into  $J$  groups. The data should be continuous observations  $(Y_1, \dots, Y_J)$ , where each  $Y_j = (y_{1,j}, \dots, y_{n_j,j})$  contains the  $n_j$  observations from group  $j$ , for  $j = 1, \dots, J$ . The function takes as input the data as a numeric vector  $\mathbf{y}$  in this concatenated form. Hence  $\mathbf{y}$  should be a vector of length  $n_1 + \dots + n_J$ . The group parameter is a numeric vector of the same size as  $\mathbf{y}$  indicating the group membership for each individual observation. Notice that with this specification the observations in the same group need not be contiguous as long as the correspondence between the variables  $\mathbf{y}$  and group is maintained.

### Model

The data are modeled using a Gaussian likelihood, where both the mean and the variance are observational-cluster-specific, i.e.,

$$y_{i,j} \mid M_{i,j} = l \sim N(\mu_l, \sigma_l^2)$$

where  $M_{i,j} \in \{1, \dots, L\}$  is the observational cluster indicator of observation  $i$  in group  $j$ . The prior on the model parameters is a Normal-Inverse-Gamma distribution  $(\mu_l, \sigma_l^2) \sim NIG(m_0, \tau_0, \lambda_0, \gamma_0)$ , i.e.,  $\mu_l \mid \sigma_l^2 \sim N(m_0, \sigma_l^2/\tau_0)$ ,  $1/\sigma_l^2 \sim \text{Gamma}(\lambda_0, \gamma_0)$  (shape, rate).

### Clustering

The model performs a clustering of both observations and groups. The clustering of groups (distributional clustering) is provided by the allocation variables  $S_j \in \{1, 2, \dots\}$ , with

$$Pr(S_j = k \mid \dots) = \pi_k \quad \text{for } k = 1, 2, \dots$$

The distribution of the probabilities is  $\{\pi_k\}_{k=1}^{\infty} \sim GEM(\alpha)$ , where GEM is the Griffiths-Engen-McCloskey distribution of parameter  $\alpha$ , which characterizes the stick-breaking construction of the DP (Sethuraman, 1994).

The clustering of observations (observational clustering) is provided by the allocation variables  $M_{i,j} \in \{1, \dots, L\}$ , with

$$Pr(M_{i,j} = l \mid S_j = k, \dots) = \omega_{l,k} \quad \text{for } k = 1, 2, \dots; l = 1, \dots, L.$$

The distribution of the probabilities is  $(\omega_{1,k}, \dots, \omega_{L,k}) \sim \text{Dirichlet}_L(\beta/L, \dots, \beta/L)$  for all  $k = 1, 2, \dots$ . Here, the dimension  $L$  is fixed.

## Value

variational\_fiSAN returns a list of class SANvb containing four objects:

- `model`: name of the fitted model.
- `params`: list containing the data and the parameters used in the simulation. Details below.
- `sim`: list containing the simulated values (optimized variational parameters). Details below.
- `time`: total computation time.

**Data and parameters:** `params` is a list with the following components:

`y`, `group`, `Nj`, `J` Data, group labels, group frequencies, and number of groups.

K, L Number of fitted distributional and observational clusters.

m0, tau0, lambda0, gamma0 Model hyperparameters.

epsilon, seed The threshold controlling the convergence criterion and the random seed adopted to replicate the run.

(hyp\_alpha1, hyp\_alpha2) or alpha Hyperparameters on  $\alpha$  (if  $\alpha$  random); or provided value for  $\alpha$  (if fixed).

beta\_bar the hyperparameter governing all the finite Dirichlet distributions at the observational level.

**Simulated values:** sim is a list with the following components:

theta\_l Matrix of size (L,4). Each row is a posterior variational estimate of the four normal-inverse gamma hyperparameters.

Elbo\_val Vector containing the values of the ELBO.

XI A list of length J. Each element is a matrix of size (N, L) posterior variational probability of assignment of assignment of the i-th observation in the j-th group to the l-th OC, i.e.,  $\hat{\xi}_{i,j,l} = \hat{\mathbb{Q}}(M_{i,j} = l)$ .

RHO Matrix of size (J, K). Each row is a posterior variational probability of assignment of the j-th group to the k-th DC, i.e.,  $\hat{\rho}_{j,k} = \hat{\mathbb{Q}}(S_j = k)$ .

a\_tilde\_k, b\_tilde\_k Vector of updated variational parameters of the Beta distributions governing the distributional stick-breaking process.

beta\_bar\_lk Matrix of updated variational parameters of the Dirichlet distributions governing the observational clustering (arranged by column).

conc\_hyper If the concentration parameters is chosen to be random, these object contain a vector with the two updated hyperparameters.

alpha If the concentration parameters is chosen to be fixed, this object contains the passed values.

## Examples

```
set.seed(1234)
y <- c( rnorm(100) , rnorm(100,5))
g <- rep( 1:2, rep(100,2))
est <- variational_fiSAN( y, g, verbose = FALSE, epsilon = 1e-2)
```

---

variational\_fSAN

*Mean Field Variational Bayes estimation of fSAN*

---

## Description

variational\_fSAN is used to perform posterior inference under the finite shared atoms nested (fSAN) model with Gaussian likelihood (originally proposed in D'Angelo et al., 2023). The model uses finite Dirichlet mixtures for both the distributional and observational levels of the model.

**Usage**

```
variational_fSAN(y, group, maxL = 30, maxK = 20,
                 m0 = 0, tau0 = .01, lambda0 = 3, gamma0 = 2,
                 alpha_bar = .005, beta_bar = .005,
                 epsilon = 1e-6, seed = NULL, maxSIM = 1e5,
                 warmstart = TRUE, verbose = FALSE)
```

**Arguments**

y	Numerical vector of observations (required).
group	Numerical vector of the same length of y, indicating the group membership (required).
maxL, maxK	integers, the upper bounds for the observational and distributional clusters to fit, respectively
m0, tau0, lambda0, gamma0	Hyperparameters on $(\mu, \sigma^2) \sim NIG(m_0, \tau_0, \lambda_0, \gamma_0)$ .
alpha_bar	the hyperparameter of the symmetric distributional Dirichlet distribution.
beta_bar	the hyperparameter of the symmetric observational Dirichlet distribution.
epsilon	the tolerance that drives the convergence criterion adopted as stopping rule
seed	random seed to control the initialization.
maxSIM	the maximum number of CAVI iteration to perform.
warmstart	logical, if TRUE, the observational means of the cluster atoms are initialized with a k-means algorithm.
verbose	logical, if TRUE the iterations are printed.

**Details****Data structure**

The finite common atoms mixture model is used to perform inference in nested settings, where the data are organized into  $J$  groups. The data should be continuous observations  $(Y_1, \dots, Y_J)$ , where each  $Y_j = (y_{1,j}, \dots, y_{n_j,j})$  contains the  $n_j$  observations from group  $j$ , for  $j = 1, \dots, J$ . The function takes as input the data as a numeric vector  $y$  in this concatenated form. Hence  $y$  should be a vector of length  $n_1 + \dots + n_J$ . The group parameter is a numeric vector of the same size as  $y$  indicating the group membership for each individual observation. Notice that with this specification the observations in the same group need not be contiguous as long as the correspondence between the variables  $y$  and  $group$  is maintained.

**Model**

The data are modeled using a Gaussian likelihood, where both the mean and the variance are observational-cluster-specific, i.e.,

$$y_{i,j} \mid M_{i,j} = l \sim N(\mu_l, \sigma_l^2)$$

where  $M_{i,j} \in \{1, \dots, L\}$  is the observational cluster indicator of observation  $i$  in group  $j$ . The prior on the model parameters is a Normal-Inverse-Gamma distribution  $(\mu_l, \sigma_l^2) \sim NIG(m_0, \tau_0, \lambda_0, \gamma_0)$ , i.e.,  $\mu_l \mid \sigma_l^2 \sim N(m_0, \sigma_l^2/\tau_0)$ ,  $1/\sigma_l^2 \sim \text{Gamma}(\lambda_0, \gamma_0)$  (shape, rate).

### Clustering

The model performs a clustering of both observations and groups. The clustering of groups (distributional clustering) is provided by the allocation variables  $S_j \in \{1, \dots, K\}$ , with

$$Pr(S_j = k | \dots) = \pi_k \quad \text{for } k = 1, \dots, K.$$

The distribution of the probabilities is  $(\pi_1, \dots, \pi_K) \sim \text{Dirichlet}_K(\alpha/K, \dots, \alpha/K)$ . Here, the dimension  $K$  is fixed.

The clustering of observations (observational clustering) is provided by the allocation variables  $M_{i,j} \in \{1, \dots, L\}$ , with

$$Pr(M_{i,j} = l | S_j = k, \dots) = \omega_{l,k} \quad \text{for } k = 1, \dots, K; l = 1, \dots, L.$$

The distribution of the probabilities is  $(\omega_{1,k}, \dots, \omega_{L,k}) \sim \text{Dirichlet}_L(\beta/L, \dots, \beta/L)$  for all  $k = 1, \dots, K$ . Here, the dimension  $L$  is fixed.

### Value

variational\_fSAN returns a list of class SANvb containing four objects:

- `model`: name of the fitted model.
- `params`: list containing the data and the parameters used in the simulation. Details below.
- `sim`: list containing the simulated values (optimized variational parameters). Details below.
- `time`: total computation time.

**Data and parameters:** `params` is a list with the following components:

`y`, `group`, `Nj`, `J` Data, group labels, group frequencies, and number of groups.

`K`, `L` Number of fitted distributional and observational clusters.

`m0`, `tau0`, `lambda0`, `gamma0` Model hyperparameters.

`epsilon`, `seed` The threshold controlling the convergence criterion and the random seed adopted to replicate the run.

`alpha_bar`, `beta_bar` the hyperparameters governing all the finite Dirichlet distributions at the distributional and observational level.

**Simulated values:** `sim` is a list with the following components:

`theta_l` Matrix of size  $(L, 4)$ . Each row is a posterior variational estimate of the four normal-inverse gamma hyperparameters.

`Elbo_val` Vector containing the values of the ELBO.

`XI` A list of length  $J$ . Each element is a matrix of size  $(N, L)$  posterior variational probability of assignment of assignment of the  $i$ -th observation in the  $j$ -th group to the  $l$ -th OC, i.e.,  $\hat{\xi}_{i,j,l} = \hat{Q}(M_{i,j} = l)$ .

`RHO` Matrix of size  $(J, K)$ . Each row is a posterior variational probability of assignment of the  $j$ -th group to the  $k$ -th DC, i.e.,  $\hat{\rho}_{j,k} = \hat{Q}(S_j = k)$ .

`a_tilde_k`, `b_tilde_k` Vector of updated variational parameters of the Beta distributions governing the distributional stick-breaking process.

`alpha_bar_k` Vector of updated variational parameters of the Dirichlet distributions governing the distributional clustering.

`beta_bar_lk` Matrix of updated variational parameters of the Dirichlet distributions governing the observational clustering (arranged by column).

## References

D'Angelo, L., Canale, A., Yu, Z., and Guindani, M. (2023). Bayesian nonparametric analysis for the detection of spikes in noisy calcium imaging data. *Biometrics*, 79(2), 1370–1382. DOI: 10.1111/biom.13626

## Examples

```
set.seed(123)
y <- c(rnorm(50), rnorm(50, 5))
g <- rep(1:2, rep(50, 2))
est <- variational_fSAN(y, g, verbose = FALSE,
                       epsilon = 1e-2, maxL=15, maxK=10)
```

---

```
variational_multistart
```

*Perform variational inference using multiple starting points.*

---

## Description

`variational_multistart` is the main function of the package. It is used to estimate via variational inference the three models we present (CAM, fiSAN, fSAN) while adopting multiple random starting points to better explore the variational parameter space. The run that provides the highest Expected Lower BOund (ELBO) is usually the one considered for inference. Note that the arguments passed to this functions are a union of the arguments of the functions `variational_CAM`, `variational_fiSAN`, and `variational_fSAN`.

## Usage

```
variational_multistart(y, group, runs, cores = 1,
                      model = c("fiSAN", "CAM", "fSAN"),
                      maxL = 30, maxK = 20,
                      m0 = 0, tau0 = .01, lambda0 = 3, gamma0 = 2,
                      conc_par = NULL, conc_hyperpar = c(1, 1, 1, 1),
                      alpha_bar = 0.05, beta_bar = 0.05,
                      epsilon = 1e-6, root = 1234, maxSIM = 1e5,
                      warmstart = TRUE)

## S3 method for class 'multistart'
plot(x, type = c("elbo", "time"), log_scale_iter = TRUE, ...)

## S3 method for class 'multistart'
print(x, ...)
```

**Arguments**

y	vector of observations.
group	vector of the same length of y indicating the group membership (numeric).
runs	the number of multiple runs to launch.
cores	the number of cores to dedicate to the multiple runs.
model	a string specifying the model to use. It can be "fiSAN", "CAM", or "fSAN".
maxL, maxK	integers, the upper bounds for the observational and distributional clusters to fit, respectively.
m0, tau0, lambda0, gamma0	hyperparameters on $(\mu, \sigma^2) \sim NIG(m_0, \tau_0, \lambda_0, \gamma_0)$ .
conc_hyperpar, conc_par, alpha_bar, beta_bar	these values crucially depend on the chosen model. See <a href="#">variational_CAM</a> , <a href="#">variational_fiSAN</a> , <a href="#">variational_fSAN</a> for proper explanations.
epsilon	the tolerance that drives the convergence criterion adopted as stopping rule.
root	common part of the random seeds used to control the initialization in order to provide reproducibility even in paralleled settings.
maxSIM	the maximum number of CAVI iteration to perform.
warmstart	logical, if TRUE, the observational means of the cluster atoms are initialized with a k-means algorithm.
x	an object of class multistart, obtained from the <code>variational_multistart</code> function.
type	a string specifying the type of plot. It can be either "elbo" or "time". The former displays the elbo trajectories, highlighting the best run. The latter provides a summary of the computational times.
log_scale_iter	logical. If TRUE, when plotting the elbo trajectories, the x-axis is displayed in log-scale, enhancing the visualization of the results.
...	ignored

**Details**

For the details of the single models, see their specific documentations: [variational\\_CAM](#), [variational\\_fiSAN](#), and [variational\\_fSAN](#).

**Value**

A list with all the runs performed. Each element of the list is a fitted variational model of class SANvb.

**See Also**

[variational\\_CAM](#), [variational\\_fiSAN](#), [variational\\_fSAN](#), [extract\\_best](#).



**Examples**

```
# Generate example data
set.seed(123)
y <- c(rnorm(100), rnorm(100, 5))
g <- rep(1:2, rep(100, 2))

# Estimate multiple models via variational inference
est <- variational_multistart(y, g, runs=5)
```

# Index

estimate\_atoms\_weights\_vi, [2](#), [2](#), [4](#)  
estimate\_clustering\_vi, [3](#)  
extract\_best, [3](#), [4](#), [5](#), [16](#)

plot.multistart  
    (variational\_multistart), [15](#)  
plot.SANvb, [5](#)  
plot.vi\_atoms\_weights  
    (estimate\_atoms\_weights\_vi), [2](#)  
plot.vi\_clustering  
    (estimate\_clustering\_vi), [3](#)  
print.multistart  
    (variational\_multistart), [15](#)  
print.SANvb, [6](#), [6](#)  
print.vi\_atoms\_weights  
    (estimate\_atoms\_weights\_vi), [2](#)  
print.vi\_clustering  
    (estimate\_clustering\_vi), [3](#)

variational\_CAM, [2-4](#), [6](#), [7](#), [16](#)  
variational\_fiSAN, [2-4](#), [6](#), [10](#), [16](#)  
variational\_fSAN, [2-4](#), [6](#), [12](#), [16](#)  
variational\_multistart, [15](#)