Package ‘Pade’

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Type Package

Title Padé Approximant Coefficients

Version 1.0.7

Date 2024-06-19

Description Given a vector of Taylor series coefficients of sufficient length as input, the function returns the numerator and denominator coefficients for the Padé approximant of appropriate order (Baker, 1975) <ISBN:9780120748556>.

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Imports utils

Suggests covr, tinytest

URL https://github.com/aadler/Pade

BugReports https://github.com/aadler/Pade/issues

Encoding UTF-8

NeedsCompilation no

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Contents

`Pade-package` *Padé Approximant Coefficients*

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Details

The DESCRIPTION file:

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Index of help topics:

```
Pade                    Padé Approximant Coefficients
Pade-package            Padé Approximant Coefficients
```

Author(s)

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Pade *Padé Approximant Coefficients*

Description

Given Taylor series coefficients a_n from $n = 0$ up to $n = T$, the function will calculate the Padé $[L/M]$ approximant coefficients so long as $L + M \leq T$.

Usage

```
Pade(L, M, A)
```

Arguments

`L`	Order of Padé numerator
`M`	Order of Padé denominator
`A`	vector of Taylor series coefficients, starting at x^0

Details

As the Taylor series expansion is the "best" polynomial approximation to a function, the Padé approximants are the "best" rational function approximations to the original function. The Padé approximant often has a wider radius of convergence than the corresponding Taylor series, and can even converge where the Taylor series does not. This makes it very suitable for computer-based numerical analysis.

The $[L/M]$ Padé approximant to a Taylor series $A(x)$ is the quotient

$$\frac{P_L(x)}{Q_M(x)}$$

where $P_L(x)$ is of order L and $Q_M(x)$ is of order M. In this case:

$$A(x) - \frac{P_L(x)}{Q_M(x)} = \mathcal{O}\left(x^{L+M+1}\right)$$

When q_0 is defined to be 1, there is a unique solution to the system of linear equations which can be used to calculate the coefficients.

The function accepts a vector `A` of length `T + 1`, composed of the a_n of the of truncated Taylor series

$$A(x) = \sum_{j=0}^{T} a_j x^j$$

and returns a list of two elements, `Px` and `Qx`, the Padé numerator and denominator coefficients respectively, as long as $L + M \leq T$.

Value

Pade returns a list with two entries:

- Px: Coefficients of the numerator polynomial starting at x^0.
- Qx: Coefficients of the denominator polynomial starting at x^0.

Author(s)

Avraham Adler <Avraham.Adler@gmail.com>

References

Baker, George Allen (1975) *Essentials of Padé Approximants* Academic Press. ISBN 978-0-120-74855-6

See Also

This package provides similar functionality to the pade function in the **pracma** package. However, it does not allow computation of coefficients beyond the supplied Taylor coefficients and it expects its input and provides its output in ascending—instead of descending—order.

See the **minimaxApprox** package for polynomial and rational minimax approximations to functions.

Examples

```
A <- 1 / factorial(0:10) ## Taylor sequence for e^x up to x^{10} around x_0 = 0
Z <- Pade(5, 5, A)
print(Z)                          ## Padé approximant of order [5 / 5]
x <- -.01                         ## Test value
Actual <- exp(x)                  ## Proper value
print(Actual, digits = 16)
Estimate <- sum(Z[[1L]] * x ^ (seq_along(Z[[1L]]) - 1)) /
    sum(Z[[2L]] * x ^ (seq_along(Z[[2L]]) - 1))
print(Estimate, digits = 16)       ## Approximant value
all.equal(Actual, Estimate)
```

Index